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How to Solve Paradoxes: A Taxonomy and Analysis of Solution-Types

Cómo resolver paradojas: Una taxonomía y el análisis de soluciones tipo

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Abstract: Just as philosophical paradoxes cluster in categories, such as the paradoxes of self-reference, justification, and so on, so too do solutions. And just as paradoxes (e.g., the liar paradox) take on new spins, get strengthened, and reappear from time to time, so too do solutions. Solution-types can be given a fairly complete taxonomy. By "solution-type" I will mean a "strategy for analyzing paradoxes." In this article, a taxonomy for solution-types to philosophical paradoxes is given. Such a taxonomy suggests that, even for the most restricted of solution-types, in which the paradox is taken to expose some kind of fundamental, unresolvable conceptual glitch, all solution types address the underlying intuitions that motivate accepting the parts of the paradox. In addition, an analysis of the taxonomy suggests that only the most shallow of philosophical paradoxes get straightforward solutions.

Keywords: Paradox, solution, solution-type, intuition, sorites.

Resumen: Tal como las paradojas encajan en categorías, tales como las paradojas de auto-referencia, justificación, y otras similares, así también las soluciones caben en categorías. Y tal como las paradojas toman nuevas formas, fuerza renovada y reaparecen de tiempo en tiempo (como la del mentiroso), así también las soluciones. Soluciones-tipo pueden dar una taxonomía muy adecuada. Por "solución-tipo" me referiré a "una estrategia para el análisis de paradojas". En este articulo, se ofrece una taxonomía de soluciones-tipo de paradojas filosóficas. Esta taxonomía sugiere que, incluso para aquellas con mayor restricción para solucione tipos en las que las paradojas son tomadas como exponentes de un tipo de concepto fundamental, todas las soluciones tipos se dirigen a la intuiciones implícitas que motivan la aceptación de partes de la paradoja. Además, el análisis de la taxonomía sugiere que sólo las más superficiales paradojas filosóficas tienen soluciones directas.

Palabras clave: Paradoja, solución, solución-tipo, intuición, sorites.

Introduction

There are many more proposed solutions to paradoxes than there are paradoxes themselves. Indeed, a glance through the long history of philosophical paradoxes reveals not only that paradoxes cluster in categories, such as the paradoxes of self-reference, justification, and so on, but that types of solutions do this, as well. Moreover, just as paradoxes (e.g., the liar paradox) take on new spins, get strengthened, and reappear from time to time, so too do solutions. Solution-types, too, can be given a fairly complete taxonomy. By "solution-type" I will mean a "strategy for analyzing paradoxes." Two theories may give different solutions to the same, or even different paradoxes, yet these two solutions may involve using the same strategy for analyzing the paradox, and hence be part of the same solution-type. For example, popular solutions for both the barber paradox (concerning the barber who shaves all and only those men that don't shave themselves) and Russell's paradox (concerning the set of all sets that don't contain themselves as members) both deny the existence of any such paradoxical entity (i.e., barber or set). Both solutions are simple denials of the entity giving rise to the paradox, the most basic strategy for solving paradoxes. Below, a taxonomy for solution-types to philosophical paradoxes is given. Such a taxonomy suggests that, even for the most restricted of solution-types, in which the paradox is taken to expose some kind of fundamental, unresolvable conceptual glitch, all solution types address the underlying intuitions that motivate accepting the parts of the paradox. In addition, an analysis of the taxonomy suggests that only the most shallow of philosophical paradoxes get straightforward solutions.

The Intuitive Basis of Paradoxes

Before turning to solutions, it will be helpful to briefly discuss paradoxes, the entities for which the solutions are proposed. Conceived very broadly, a *paradox* can be anything from a tough problem, to a counterintuitive opinion or conclusion. Yet, while philosophers are by no means in complete agreement of the correct way to define *paradox*, within philosophical circles paradoxes are generally something more specific. On one philosophical definition, a *paradox* is *an argument with seemingly valid reasoning and true premises, but an obviously false conclusion* (cf. Mackie). Another common definition (cf. Resnick) holds that a *paradox* is *a set of mutually inconsistent propositions, each of which seems true*. Still others claim that *paradoxes* are *unacceptable conclusions drawn from seemingly true premises and correct reasoning* (cf. Sainsbury). What all philosophical definitions have in common is that paradoxes engender strong intuitions about the truth-values of propositions and about the reasoning in the paradox. And since the parts of the paradox are in conflict, those who attempt to solve them are often called on to explain why the strong intuition that we have regarding one part of the paradox is mistaken. In this way, these definitions are faithful to the etymological roots of the term *paradox*, which comes from the Greek terms for *against* or *beyond* (*para*) and *expectation* or *opinion* (*doxa*). The Greek terms emphasize the counterintuitive nature of paradoxes.

To illustrate this, consider two examples of paradoxes, which for uniformity's sake, I'll formulate as arguments:

Sorites Paradox

1. A person with 0 hairs is bald.
2. For any number n, if a person with n hairs is bald, then a person with (n + 1) hairs is bald.
3. Therefore, a person with 1,000,000 hairs is bald.

Russell's Paradox

Let R be the class of all classes that are not members of themselves.

1. For any object x, x is a member of R if, and only if it is not the case that x is a member of itself.
2. R is a member of R, if, and only if it is not the case that R is a member of itself.

3. Therefore, R is a member of R if, and only if it is not the case that R is a member of R.

Or, simply:

1. For any class *x, x* Œ R iff ÿ *x* Œ *x.*
2. R Œ R iff ÿ R ŒR

In these arguments we have intuitively plausible premises, apparently correct reasoning and an obviously false or contradictory conclusion. With regards to the sorites paradox, the first premise, which claims that a person with no hairs is bald, describes the paradigm case of baldness. Such a premise seems unobjectionable, since if any person were to possess the property of baldness, the person with the least possible numbers of hair would. The second premise is very intuitive as well. It claims that the difference of one hair is not enough to warrant the change in classification from being bald to being non-bald. With regards to its reasoning, the sorites is straightforward. The first premise claims that a person with a specific number of hairs (0) is bald. The second premise makes a claim about all numbers of hairs, saying that for any arbitrary number, one more or one less would not make enough of a difference to warrant a change in classification from someone's being bald to non-bald, or vice versa. The number used in premise one is plugged into the generalization in the second premise to get the conclusion that a person with a million hairs is bald.

In the case of Russell's paradox, we have fairly straightforward premise and a contradictory conclusion. Although the condition of R may be somewhat hard to read, there is no *prima facie* problem with a class of classes that do not contain themselves as members. There are classes, it seems, that do contain themselves as members. The class of classes with more than 2 members is, it seems, a member of itself. In addition, there are many classes that do not contain themselves as members. For example, the class of books is not a book and is therefore not a member of itself. So why not a class of classes that are not do not contain themselves as members? The condition for R is licensed by Cantorian, "naïve" set theory's principle of abstraction. The principle of abstraction holds that "A formula P (*x*) defines a set A by the convention that the members of A are exactly those objects a such that P

(a) is a true statement." That is, a formula is the defining property of a set if, and only if, all and only members of that set satisfy the formula. As a result, every property determines a set. For properties such as being a round square, the set is empty. And so the property of being a non-self membered class, it follows, determines a class as well. Next, R replaces the *x* in (1), and this leads to the contradictory conclusion.

Whether formulated as arguments as in the above cases, or as sets of propositions that are in conflict, the sorites and Russell's paradoxes are paradoxical in that they engender strong intuitions about propositions that are contradictory.

Solution Types: Early Treatments

Since Aristotle, solutions to paradoxes have been commonly thought to be the finding flaws in the paradoxical argument (cf. Kneale and Kneale.) In addition, recent analyses of solution-types were given by Charles Chihara and Stephen Schiffer. Chihara claimed that there are two main problems that a proposed solution to a paradox must solve (i) the diagnostic problem of explain what leads to the paradox, and (ii) the preventative problem of creating a logical system on which the paradox does not arise (1979). For example, consider the simple liar paradox. Take simple liar sentence, L: *This sentence is false*. If L if false, it is true. But if L is true, then it is false. And given that any statement is either true or false (bivalence), it must be one of the two. Hence it is both true and false. A solution given by a logic with truth-value gaps would solve the diagnostic problem by pointing to the principle of bivalence. The account would then introduce another logical system on which this is rectified, and L would be interpreted as neither true nor false. While Chihara's account does include many solutions such as this one, many other solutions to paradoxes do not provide a preventative solution to the paradox. Consider Michael Dummett's solution to the sorites paradox. Dummett provides a restricted solution, ultimately showing how the paradox arises, but concluding that no preventative solution can be given. Thus, solutions, which we are thinking of as strategies for analyzing paradoxes, may not always provide preventative measures to avoid paradoxes.

Stephen Schiffer has also given a brief analysis of solution-types in his

"Two Issues of Vagueness," distinguishing between happy-face and unhappy-face solutions to paradoxes. According to Schiffer:

> A *happy-face solution* to a paradox would do two things: first, it would identify the odd-guy-out, the seemingly true proposition that isn't really true; and second, it would remove from this proposition the air of seeming truth so that we could clearly see it as the untruth it is (20, italics Schiffer's).

Unlike happy-face solutions, unhappy-face solutions do not attempt to expose a seemingly true part of the paradox for the untruth that the part is. Instead such solutions indicate what about the relevant notion leads to paradox. In addition, such solutions may propose an alternative notion, one which does all that is needed of the original notion but does not lead to paradox. For example, Tarski gave an unhappy-face solution to liar paradox. According to Tarski, the ordinary notion of truth is incoherent and leads to the liar paradox, but a new notion of truth could be devised that does not lead to paradox. Schiffer's account points to the importance of philosophical intuitions for both generating paradox and finding solutions. Building on this, the account given below presents a more fine-grained analysis of solution-types.

Taxonomy of Solution-Types

Given the importance of philosophical intuition in the generation of paradox, solutions cluster in different approaches to the intuitions that make the parts of the paradox so plausible. The taxonomy of solution-types given below, like any taxonomy of paradoxes, has an element of artificiality. Some solutions occupy border regions between types. Just as Russell's paradox is both a set theoretic paradox and a paradox of self-reference, solutions, too, may not fit exactly into some particular taxonomy. I will restrict my examples to one or two examples of each solution type.

Solution Type 1: Denying the Existence of the Paradoxical Entity

When confronted with a paradox we can deny that there is any such paradoxical entity, thus side-stepping the paradox altogether. In such a solution, we don't point to any flaw in the argument, other than the fact that one of the terms is vacuous. Consider the barber paradox. In a remote village in Sicily, there is a barber that shaves all and only the men of the town who do not shave themselves. Who, then, shaves the barber? If he shaves himself, then he doesn't need his services. But if he doesn't shave himself, then he does. So the barber both shaves and does not shave himself. Mark Sainsbury summarizes the solution this way:

> The unacceptable supposition is that there is such a barber—one who shaves himself if, and only if, he does not. The story may have sounded acceptable; It turned our minds, agreeably enough, to the mountains of inland Sicily. However, once we see what the consequences are, we realize that the story cannot be true: There cannot be such a barber or such a village. The story is unacceptable. This is not a very deep paradox because the unacceptability is very thinly disguised by the mountains and the remoteness (2).

To Sainsbury, our intuitions about the barber are wrong, because we are lulled into believing that there can be things like barbers that shave all an only the men of the town who do not shave themselves.

Solution Type 2: Denying an Assumption

By far the most common strategy for solving paradoxes is to point to an assumption made by the paradox and show that it is false, and that our intuitions regarding the plausibility of that assumption are misleading. Consider a standard solution to the simple unexpected examination paradox. A teacher announces that there will be a surprise exam one day next week, but a student presents a proof that this cannot be. If the exam is held on Friday, then there will be no element of surprise, because all the other possible days have been eliminated. So Friday is ruled out. If the exam is held Thursday then there will be no surprise either, because Friday has been ruled out, as well. The same goes for the other days of the week, including Monday. Monday is the last remaining option, so there is no element of surprise there either. Therefore, there can be no unexpected exam. A standard reply to this paradox is that the student's argument, which is a *reductio* of the teacher's announcement, takes as an assumption the truth of the teacher's claim, something that the student is not licensed to do. Simply supposing the teacher's announcement is true is not enough to conclude that there will be no exam on Friday. Thus, an assumption that must be made in order to motivate the paradox cannot be drawn.

Solution Type 3: Denying the Validity of the Reasoning

Such a strategy takes issue with the underlying reasoning of the paradoxical argument. Another way to think of this is that it denies that each of the seemingly true propositions are really incompatible. An example of this type of strategy comes from the contextualist solution to a skeptical paradox. Consider the following.

1. I can know for sure that I am in New York, only if I can know that I am not dreaming.
2. I cannot know that I am not dreaming.
3. I cannot know for sure that I am in New York.

A contextualist solution to this paradoxical argument involves pointing to an equivocation in the meaning of the crucial term, *know*. Standards for knowability vary in different contexts, claims the contextualist. To know that one is in New York in the ordinary sense, involves far less stringent criteria than the knowledge that hinges certainty. Thus, *know* in the first premise of the argument means something different than *knows* in the second premise. The argument is therefore invalid, according to the contextualist.

Solution Type 4: Affirming the Conclusion

A less common strategy is to show that the conclusion, though seemingly false, is in fact true. The task then is to show why the conclusion is acceptable, despite all appearances to the contrary. A prominent case of this is the account of the liar paradox given by dialetheism, the view that some contradictions are, in fact, true. On this account, the conjunction of both the liar sentence and its denial (L & ~L) can be accepted as true. This is typically thought to be troubling because contradictions, if taken to be true, can be used to prove anything is true[1] . To mitigate this problem, the dialetheist uses a paraconsistent logic, which abandons some very intuitive logic principles, such as disjunctive syllogism, but allows for the truth of contradiction without entailing trivialism (the view that every statement is true). (cf., Priest).

Solution Type 5: Accepting the Paradox

This type of solution affirms the intuitive basis of each part of the paradox. Each part of the paradox according to this solution-type, has strong intuitive force because each part of the paradox accurately represents some feature of the concepts that lead to the paradox. The concepts themselves lead to paradox, so the only to avoid paradox is to provide a replacement concept. This does not, however, remove the paradox. In "Semantic Conception of Truth," Tarksi gives a diagnosis of why the liar paradox arises. For Tarski, this happens because natural languages are semantically closed, that is, the same expressions of a language are used to describe the language itself. If there were a distinction between the language used (object language), and the language that describes this language (metalanguage), sentences like "This sentence is false" would not be acceptable. For Tarski, there is no way to avoid conflating the object and metalanguage using in the ordinary language notion of truth. However, he proposes another way of thinking about truth, involving satisfaction and using a semantically open language that will avoid the paradox. Tarski thus provides an alternative concept that is not meant to be a substitute for our natural language concept of truth.

[1] 1. P & ~P Assume Contradiction.
2. P 1& E
3 ~P 1 &E
4. P v A 2vI
5. A 3,4 Dis. Syllogism.

Are Certain Solution-Types More Sucessful Than Others?

I believe that, except for the most shallow of paradoxes in which a fallacy is only vaguely hidden, paradoxes admit only of the last solution-type. Every paradox has premises that seem true, otherwise the paradox would merely be an unsound argument. In addition, intuition tells us that the conclusion is false and that the reasoning is valid. As such, paradoxes force us to confront extremely strong, but conflicting intuitions about basic folk concepts that give rise to the paradoxes, such as truth, baldness, knowledge, belief, sets, and many others. The users of solution-types 1 through 4 attempt to find flaws in paradoxical arguments and then explain why we have the intuition that there is no flaw. Yet, the intuitions that we have about such folk concepts are, in fact, reliable and not to be discounted. They cannot be explained away as understandable but misleading and paradoxes cannot be treated as flawed arguments that merely look sound. This is due to the fact that the concepts that generate paradoxes, concepts like knowledge, space, truth, and so on, arise out of our own, flawed linguistic practices. Certainly new concepts can be generated on which paradoxes don't arise. However, these concepts have only a passing resemblance to the concepts that they are meant to replace.

Moreover, an influential argument in the philosophy of science is relevant to our discussion of paradoxes. It runs as follows: Throughout the long history of science most scientific theories have been proven false and the entities posited by these theories were proven not to exist. Based on this evidence from the past, it is rational to conclude that propositions of present (and future) scientific theories are false and the entities posited by the theories non-existent, as well. Larry Laudan famously provided a long list of empirically successful theories, that is, generally accepted theories that could usually provide successful predictions, that were eventually rejected and their theoretical terms shown not to refer. The list includes: the crystalline spheres of ancient and medieval astronomy, the humoral theory of medicine, the effluvial theory of static electricity, catastrophic geology and its commitment to a universal (Noachian) flood, the phlogiston theory of heat, the vibratory theory of heat, the vital force theory of physiology, the theory of circular inertia, theories of spontaneous generation, the optical ether theory, the electromagnetic ether theory, and many others.

This line of argument, the pessimistic meta-induction argument, can be used to question not only realist theories of science, but other types of theory, including theories that provide would-be solutions to philosophical paradoxes. Take, as an example, the sorites paradox. In the approximately 2000 years since the paradox was first discussed, there have countless attempts to provide a straightforward solution to it. And although each present-day advocate of a particular solution claims to have solved the paradox, it stubbornly refuses such solution. Moreover, there is even more of a lack of consensus in the case of solutions to paradoxes than there is in competing scientific theories. Given the vast amount of time and effort, and given that such solutions fare even worse than false scientific theories in terms of establishing a consensus and approximating a solution, the most rational attitude to take toward the deepest paradoxes is that they lack clear-cut solutions.

A number of objections have been posed to the pessimistic metainduction, most pointing to the progress of science, the greater reliability of theories to predict results. Such types of response, while potentially applicable to the use of metainduction the realist/antirealist debate, has little value in terms of a response to the pessimistic metainduction applied to the history of philosophical paradoxes. There is far less appreciable progress in the history of solutions to paradoxes. In fact, some of the same solutions posited by the Ancients are still posited today. Moreover, there is nothing like the consensus of acceptance that we see for scientific theories in solutions to philosophical paradoxes.

Admittedly, a pessimistic metainduction does not provide reasons why paradoxes, qua paradoxes, lack solution-types 1-4. Nor could the conclusions drawn by this type of reasoning be decisive in the sense of a successful deductive proof. However, what such an argument does do is provide grounds for thinking a particular research strategy probably fruitless. To draw a parallel to other contexts, the mind-body problem, problems of personal identity, and others in the history of philosophy were not "solved" in a standard sense. The terms of the debate were changed. Does such an argument provide decisive grounds for rejecting solution-types 1-4 for all but the most shallow paradoxes? If by "decisive" we mean an exceedingly strong probability that such solutions will fail, then yes. If by "decisive," we mean irrefutable logical proof, then of course, no. But coupled with the conceptual argument presented earlier, it strikes me as exceedingly implausible to claim that philosophical paradoxes have such solutions.

Conclusion

Philosophical paradoxes have received countless pages of treatment by some of the best philosophers over thousands of years. The claim that such timeless problems lack all but the most restricted solution-type will strike many as an extremely pessimistic view about the philosophical enterprise. Philosophical paradoxes are about as old as philosophy itself and are like old friends to many philosophers, myself included. But, as Aristotle once said, we must prefer the truth to friends. Paradoxes lack solution-types 1-4 because paradoxes expose conceptual glitches in the folk concepts that give rise to them. How, then should paradoxes be solved? To best "solve" any but the most shallow of philosophical paradoxes requires accepting that they expose fundamental flaws in the concepts that lead to paradox. Alternative concepts may then be introduced, and these concepts may prevent some of the negative consequence that were implied by the original paradox, but there are no straightforward solutions in the sense of pointing to fallacies in the paradox. A system may be constructed on which premises are false, conclusions true or the reasoning is invalid, but this involves creating a substitute concept different from the one that led to the paradox, and an ultimate acceptance of the paradox.

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